Logic

Propositional calculus

- Propositions may be true or false, but not both:

it's raining

3+2=5

it's raining ***and*** *it's sunday*

- Tautology: the proposition is <u>always</u> true.

in Szczecin there are lions ***or*** *in Szczecin there are no lions*

Logic

Propositional calculus

- Contradiction: the proposition is always false.

in Szczecin there are lions ***and*** *in Szczecin there are no lions*

Logic

Propositional calculus

- Negation (NOT): $\neg p$

*(it's **not** raining)* $\neg(\neg p) = p$

- Conjunction (AND): $p \wedge q$

(*it's raining **and** it's sunday*)

- Disjunction (OR): $p \vee q$

*(it's raining **or** it's windy)*

- Exclusive disjunction (XOR): $p \otimes q$

*(I travel **either** by car **or** by train) (not both)*

Logic

Propositional calculus

- Conditional: $p \rightarrow q$

  *(**if** it rains **then** the street is wet)*

- Biconditional: $p \leftrightarrow q$
- True: 1, T
- False: 0, $\bot$, F

Logic

Propositional calculus - Table

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \otimes q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	F	T	T	F	T	T
T	F	F	F	T	T	F	F
F	T	T	F	T	T	T*	F
F	F	T	F	F	F	T*	T

Logic

Propositional calculus – Example

If *Kazio comes* ***then*** *we play football,*

and

If *Kazio does* ***not*** *come* ***then*** *we go to the cinema*

S: $(p \rightarrow q) \land ((\neg p) \rightarrow r)$

Logic

Propositional calculus – Example

S: (p→q) Λ ((¬p)→r)

p	q	r	p→q	¬p	(¬p)→r	S
T	T	T	T	F	T*	T
T	T	F	T	F	T*	T
T	F	T	F	F	T*	F
T	F	F	F	F	T*	F
F	T	T	T*	T	T	T
F	T	F	T*	T	F	F
F	F	T	T*	T	T	T
F	F	F	T*	T	F	F

Logic

Propositional calculus – Examples

$$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$$

$$(p \wedge q) \rightarrow \neg p$$

$$[((p \rightarrow q) \wedge r) \vee ((\neg p \wedge \neg t) \rightarrow \neg u)] \rightarrow [(\neg q \vee \neg t) \rightarrow p]$$

Logic

Propositional calculus

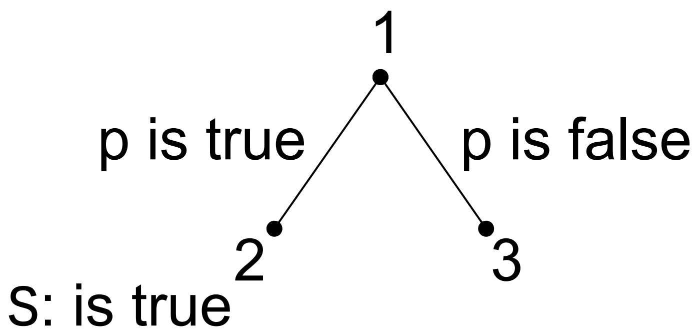
The tables may have too many columns. Then we may apply semantic trees or validation by refutation.

Logic

Propositional calculus – Semantic Trees

S: (p→q) → (¬p→¬q)

Node 2: T T* FT T*

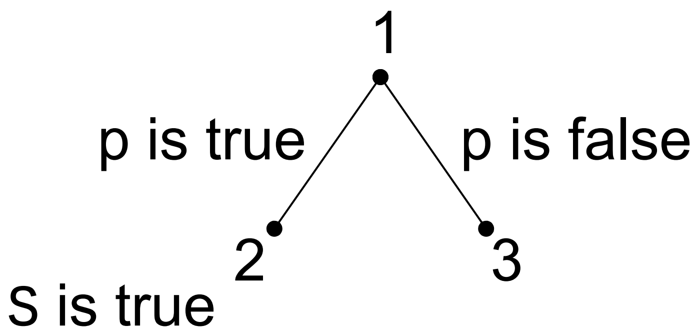


Logic

Propositional calculus – Semantic Trees

S: (p→q) → (¬p→¬q)

Node 3: F TF

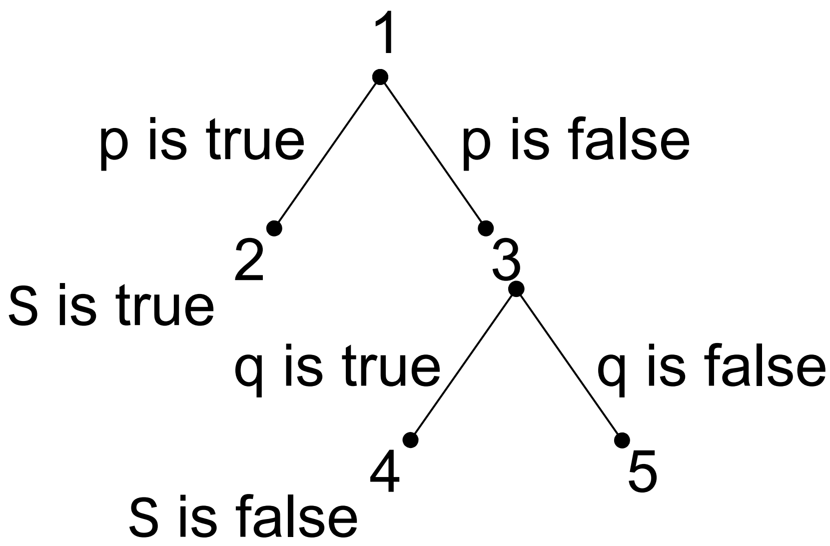


Logic

Propositional calculus – Semantic Trees

S: $(p \to q) \to (\neg p \to \neg q)$

Node 4: F T F TF F FT

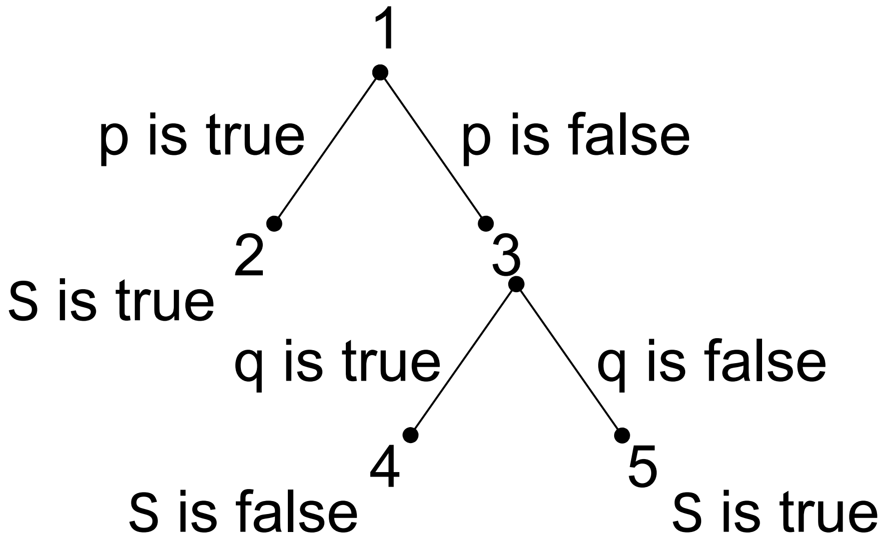


Logic

Propositional calculus – Semantic Trees

S: $(p \rightarrow q) \rightarrow (\neg p \rightarrow \neg q)$

Node 5: F F T* TF T TF

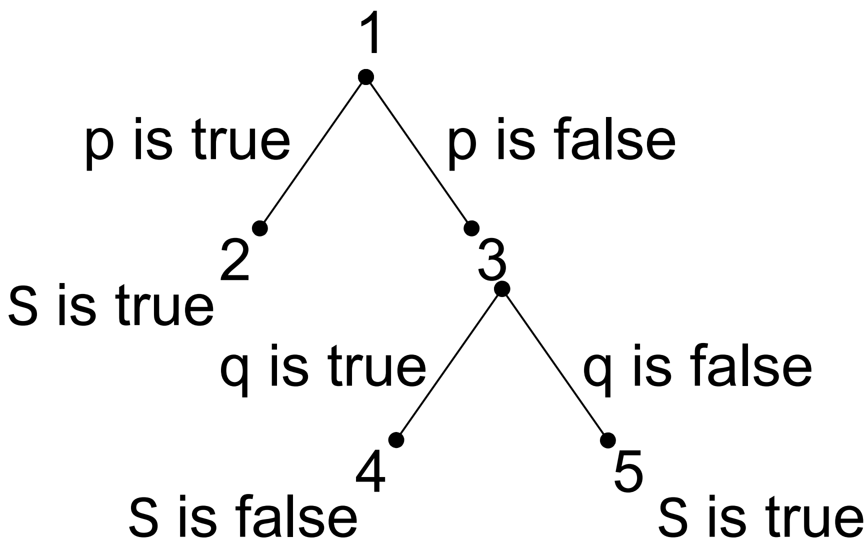


Logic

Propositional calculus – Semantic Trees

S: $(p \rightarrow q) \rightarrow (\neg p \rightarrow \neg q)$

<u>S is not a tautology</u> (= is not valid): it is false for node 4



Logic

Propositional calculus – Refutation

$$S: \ (\neg p \lor \neg q) \rightarrow \neg(p \land q)$$

We assume that the sentence S is false and then check if this is a contradiction.

Logic

Propositional calculus – Refutation

```
S:  (¬p∨¬q) → ¬(p∧q)
        T      F   F T T T
     FT F FT
```

We step into a contradiction when we assume that the sentence is false. The antecedent must be true and false at the same time. So, the sentence S has to be valid (is a tautology).